Exercise 48

Find the limit.

$$\lim_{x\to 0}\frac{\sin(x^2)}{x}$$

Solution

Rewrite the limit in terms of one that is known.

$$\begin{aligned}\lim_{x\to 0}\frac{\sin(x^2)}{x}=\lim_{x\to 0}x\cdot\frac{\sin(x^2)}{x^2}&=\left(\lim_{x\to 0}x\right)\left[\lim_{x\to 0}\frac{\sin(x^2)}{x^2}\right]\\&=\left(\lim_{x\to 0}x\right)\left(\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\right)\\&=(0)(1)\\&=0\end{aligned}$$